## Chapter 4

4.2

X has probability mass function given by

a)

| a | -1 | 0 | 1 | 2 |
|---|---|---|---|---|
| p(a) | .25 | 0.125 | 0.125 | 0.5 |

Y = X*X, so Y takes on values 0, 1, and 2. Its probability mass function is given by

| b | 0 | 1 | 2 |
|---|---|---|---|
| p(b) | 0.125 | 0.375 | 0.5 |

This happens because there are two values of X which yield Y=1.

b)

For X, the probability distribution function is

| | |
|---|---|
| 0 | $x < -1$ |
| 0.25 | $-1 \leq x < 0$ |
| 0.375 | $0 \leq x < 1$ |
| 0.5 | $1 \leq x < 2$ |
| 1.0 | $2 \leq x$ |

For Y, the probability distribution function is

| | |
|---|---|
| 0 | $x < 0$ |
| 0.125 | $0 \leq x < 1$ |
| 0.5 | $1 \leq x < 2$ |
| 1.0 | $2 \leq x$ |

From these we can see that

$P(X \leq 1) = 0.5$

$P(X \leq ¾) = 0.375$

$P(X \leq 0.14159\ldots) = 0.375$

$P(Y \leq 1) = 0.5$

$P(Y \leq ¾) = 0.126$

$P(Y \leq 0.14158\ldots) = 0.125$

4.3
Given a probability distribution function

$$F(a) = \begin{cases} 0 & a < 0 \\ 1/3 & 0 \leq a < ½ \\ ½ & ½ \leq a < ¾ \\ 1 & ¾ \leq a \end{cases}$$

The mass function is
p(0) = 1/3
p(1/2) = ½ - 1/3 = 1/6
p(3/4) = 1 – ½ = ½

4.4
We toss n coins and each coin that comes down tails is re-tossed once.
X = total number of heads seen.

a) What kind of distribution does X have?

The probability that any coin ultimately shows a head is
q = p + (1-p)p the probability that the first throw was a head + the probability that the first was a tail and the second was a head.

P(X=k) = q^k * (1-q)^(1-k), a binomial distribution. Its parameters are n and q (where q is a function of p).

4.11
The probability of winning lottery 1 after k plays has a geometric distribution. The mass function is given by"
P(X1 = k) = (p1-1)^(k-1) * p1.
The mass function for lottery 2 is given by
P(X2 = k) = (p2-1)^(k-1) * p2.

The probability of winning either (or both) after k plays is
the sum of these two probabilities less their product (since winning either is independent of winning the other.

4.14
A coin, with probability p of coming down heads, is thrown until it comes up heads for a second time.
a) What are P(X=2) P(X=3) and P(X=4)
P(X=2) = p*p (the probability that it comes up heads both times)
The possibilities for three tosses are.
HTH THH

In each case, there are two H and one T, so each of these has probability p^2 * (1-p). These events are disjoint so the total probability is
P(X=3) = 2 * (p^2) * (1-p)

For four tosses, the possibilities are
HTTH THTH TTHH
This time there are two H and two T and three disjoint possibilities so
P(X=4) = 3*(p^2)* (1-p)^2

In the general case where we throw n coins, the number of heads will be 2 and the number of tails will be n-2. The second head must appear as the last toss (for after that we quit) The first head can occur in any of other (n-1) positions. All of these possible events are disjoint, so we can concluse that
P(X=n) = (n-1)*(p^2)*(1-p)^(n-2).